S4 CALIBRATION OF A GAUGE BLOCK OF NOMINAL LENGTH 50 MM

S4.1 The calibration of the grade 0 gauge block (ISO 3650) of 50 mm nominal length is carried out by comparison using a comparator and a calibrated gauge block of the same nominal length and the same material as reference standard. The difference in central length is determined in vertical position of the two gauge blocks using two length indicators contacting the upper and lower measuring faces. The actual length l_X' of the gauge block to be calibrated is related to the actual length l_S' of the reference standard by the equation

$$l_X' = l_S' + \delta l \qquad \text{(S4.1)}$$

with δl being the measured length difference. l_X' and l_S' are the lengths of the gauge blocks under measurement conditions, in particular at a temperature which, on account of the uncertainty in the measurement of laboratory temperature, may not be identical with the reference temperature for length measurements.

S4.2 The length l_X of the unknown gauge block at the reference temperature is obtained from the relationship:

$$l_X = l_S + \delta l_D + \delta l + \delta l_C - L(\bar{\alpha} \times \delta t + \delta\alpha \times \Delta\bar{t}) - \delta l_V \qquad \text{(S4.2)}$$

where:

l_S	-	length of the reference gauge block at the reference temperature t_0 = 20 °C according to its calibration certificate;
δl_D	-	change of the length of the reference gauge block since its last calibration due to drift;
δl	-	observed difference in length between the unknown and the reference gauge block;
δl_C	-	correction for non-linearity and offset of the comparator;
L	-	nominal length of the gauge blocks considered;
$\bar{\alpha} = (\alpha_X + \alpha_S)/2$	-	average of the thermal expansion coefficients of the unknown and reference gauge blocks;
$\delta t = (t_X - t_S)$	-	temperature difference between the unknown and reference gauge blocks;
$\delta\alpha = (\alpha_X - \alpha_S)$	-	difference in the thermal expansion coefficients between the unknown and the reference gauge blocks;
$\Delta\bar{t} = (t_X + t_S)/2 - t_0$	-	deviation of the average temperature of the unknown and the reference gauge blocks from the reference temperature;
δl_V	-	correction for non-central contacting of the measuring faces of the unknown gauge block.

S4.3 **Reference standard (l_S):** The length of the reference gauge block together with the associated expanded uncertainty of measurement is given in the calibration certificate of a set of gauge blocks as 50,000 02 mm ±30 nm (coverage factor k = 2).

S4.4 **Drift of the standard (δl_D):** The temporal drift of the length of the reference gauge block is estimated from previous calibrations to be zero with limits ±30 nm. General experience with gauge blocks of this type suggests that zero drift is most probable and that a triangular probability distribution can be assumed.

S4.5 **Comparator (δl_C):** The comparator has been verified to meet the specifications stated in EAL-G21. From this, it can be ascertained that for length differences D up to ±10 µm corrections to the indicated length difference are within the limits ±(30 nm +0,02·$|D|$). Taking into account the tolerances of the grade 0 gauge block to be calibrated and the grade K reference gauge block, the maximum length difference will be within ±1 µm leading to limits of ±32 nm for non-linearity and offset corrections of the comparator used.

S4.6 **Temperature corrections ($\bar{\alpha}$, δt, $\delta\alpha$, $\Delta\bar{t}$):** Before calibration, care is taken to ensure that the gauge blocks assume ambient temperature of the measuring room. The remaining difference in temperature between the standard and the gauge block to be calibrated is estimated to be within ±0,05 K. Based on the calibration certificate of the reference gauge block and the manufacturer's data for the gauge block to be calibrated the linear thermal expansion coefficient of the steel gauge blocks is assumed to be within the interval $(11{,}5\pm1{,}0)\times10^{-6}\ °C^{-1}$. Combining the two rectangular distributions the difference in linear thermal expansion coefficient is triangularly distributed within the limits $\pm2\times10^{-6}\ °C^{-1}$. The deviation of the mean temperature of measurement from the reference temperature t_0 = 20 °C is estimated to be within ±0,5 °C. The best estimates of the difference in linear expansion coefficients and the deviations of the mean temperature from the reference temperature are zero. Therefore second order terms have to be taken into account in the evaluation of their uncertainty contribution resulting in the product of standard uncertainties associated with the factors of the product term $\delta\alpha\times\Delta\bar{t}$ in equation (S4.2) (see the mathematical note in paragraph S4.13, eq. (S4.5)). The final standard uncertainty is $u(\delta\alpha\times\Delta\bar{t}) = 0{,}236\times10^{-6}$.

S4.7 **Variation in length (δl_V):** For gauge blocks of grade 0 the variation in length determined from measurements at the centre and the four corners has to be within ±0,12 µm (ISO 3650). Assuming that this variation occurs on the measuring faces along the short edge of length 9 mm and that the central length is measured inside a circle of radius 0,5 mm, the correction due to central misalignment of the contacting point is estimated to be within ±6,7 nm.

S4.8 **Correlation:** None of the input quantities are considered to be correlated to any significant extent.

S4.9 **Measurements (δl):** The following observations are made for the difference between the unknown gauge block and the reference standard, the comparator being reset using the reference standard before each reading.

obs. no.	**obs. value**
1	-100 nm
2	-90 nm
3	-80 nm
4	-90 nm
5	-100 nm

arithmetic mean: $\overline{\delta l} = -94 \text{ nm}$

pooled estimate of standard deviation: $s_p(\delta l) = 12 \text{ nm}$
(obtained from prior evaluation)

standard uncertainty: $u(\delta l) = s(\overline{\delta l}) = \frac{12 \text{ nm}}{\sqrt{5}} = 5{,}37 \text{ nm}$

The pooled estimate of the standard deviation has been taken from the tests made to confirm compliance of the comparator used with the requirements of EAL-G21.

S4.10 **Uncertainty budget (δl_X)**:

quantity X_i	estimate x_i	standard uncertainty $u(x_i)$	probability distribution	sensitivity coefficient c_i	uncertainty contribution $u_i(y)$
l_S	50,000 020 mm	15 nm	normal	1,0	15,0 nm
δl_D	0 mm	17,3 nm	triangular	1,0	17,3 nm
δl	-0,000 094 mm	5,37 nm	normal	1,0	5,37 nm
δl_C	0 mm	18,5 nm	rectangular	1,0	18,5 nm
δt	0 °C	0,0289 °C	rectangular	-575 nm°C^{-1}	-16,6 nm
$\delta\alpha \times \Delta\bar{t}$	0	$0{,}236\times10^{-6}$	special	50 mm	-11,8 nm
δl_V	0 mm	3,87 nm	rectangular	-1,0	-3,87 nm
l_X	49,999 926 mm				36,4 nm

S4.11 **Expanded uncertainty**

$$U = k \times u(l_X) = 2 \times 36{,}4 \text{ nm} \cong 73 \text{ nm}$$

S4.12 **Reported result**

The measured value of the nominal 50 mm gauge block is 49,999 926 mm ±73 nm.

The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor k = 2, which for a normal distribution corresponds to a coverage probability of approximately 95 %.

S4.13 **Mathematical note on the standard uncertainty of measurement of the product of two quantities with zero expectation:** If a product of two quantities is considered, the usual method of evaluation of uncertainty contributions based on the linearisation of the model function has to be modified if one or both of the expectations of the factors in the product are zero. If the factors in the product are statistically independent with non-zero expectations, the square of the relative standard uncertainty of measurement (relative variance) associated with the product can be expressed without any linearisation by the squares of the relative standard uncertainties associated with the estimates of the factors:

$$w^2(x_1 \times x_2) = w^2(x_1) + w^2(x_2) + w^2(x_1) \times w^2(x_2) \qquad \text{(S4.2)}$$

Using the definition of the relative standard uncertainty of measurement this expression is easily transformed into the general relation

$$u^2(x_1 \times x_2) = x_2^2\, u^2(x_1) + x_1^2\, u^2(x_2) + u^2(x_1) \times u^2(x_2) \qquad \text{(S4.3)}$$

If the standard uncertainties $u(x_1)$ and $u(x_2)$ associated with the expectations x_1 and x_2 are much smaller than the moduli of the respective expectation values the third term on the right side may be neglected. The resulting equation represents the case described by the usual method based on the linearisation of the model function.

If, however, one of the moduli of the expectation values, for example $|x_2|$, is much smaller than the standard uncertainty $u(x_2)$ associated with this expectation or even zero, the product term involving this expectation may be neglected on the right side of equation (S4.3), but not the third term. The resulting equation is

$$u^2(x_1 \times x_2) \cong x_1^2\, u^2(x_2) + u^2(x_1) \times u^2(x_2) \qquad \text{(S4.4)}$$

If both moduli of the expectation values are much smaller than their associated standard uncertainties or even zero, only the third term in equation (S4.3) gives a significant contribution:

$$u^2(x_1 \times x_2) \cong u^2(x_1) \times u^2(x_2) \qquad \text{(S4.5)}$$

S5 CALIBRATION OF A TYPE N THERMOCOUPLE AT 1000°C

S5.1 A type N thermocouple is calibrated by comparison with two reference thermocouples of type R in a horizontal furnace at a temperature of 1000 °C. The emfs generated by the thermocouples are measured using a digital voltmeter through a selector/reversing switch. All thermocouples have their reference junctions at 0 °C. The thermocouple to be calibrated is connected to the reference point using compensating cables. Temperature values are give in the t_{90} temperature scale.

S5.2 The temperature t_X of the hot junction of the thermocouple to be calibrated is

$$\begin{aligned} t_X &= t_S(V_{iS} + \delta V_{iS1} + \delta V_{iS2} + \delta V_R - \frac{\delta t_{0S}}{C_{S0}}) + \delta t_D + \delta t_F \\ &\cong t_S(V_{iS}) + C_S \times \delta V_{iS1} + C_S \times \delta V_{iS2} + C_S \times \delta V_R - \frac{C_S}{C_{S0}} \delta t_{0S} + \delta t_D + \delta t_F \end{aligned} \qquad \text{(S5.1)}$$